الجامعةالسعوديةالالكترونية
Saudi Electronic University

Ministry of Higher Education
Saudi Electronic University
College of Science and Humanities

Midterm Examination Cover Sheet

Second Semester: 1436-1437 / 2015-2016

Course Instructor:		**Exam Date:**	**27-10-2015**
Course Title:	**Linear Algebra**	**Course Code:**	**Math 251**
Exam Duration:	**One Hour**	**Number of Pages:** (including cover page)	**6**

Exam Guidelines

- **Mobile phones are not permitted.**
- **Calculators are permitted.**
- **Exchange of calculators is not allowed.**

Marking Scheme

Questions	Score
Question 1	/ 8
Question 2	/ 5
Question 3	/ 3
Question 4	/ 2
Question 5	/ 3
Question 6	/ 4
Total	/ 25

Student Name:	**Student ID:**

1. (1 mark each). State whether the following statements are true or false:

a) The vectors (2,3,5) and (5,3,2) have same magnitude. (True)

b) If A is invertible, then A^T is not invertible. (False)

c) If A is a square matrix then $det(A) \cdot det(A^{-1}) = 1$ (True)

d) For any scalar k, $k(u \cdot v) = ku + kv$. (False)

e) If A and B both are $m\ x\ n$ matrices, then both $A\ B^T$ and A^T B are defined. (True)

f) If V is any vector space and S={ $v_1, v_2, \cdots, v_n$} is a finite set of vectors in V, then S is called a basis for V if and only if S spans V. (False).

g) If a set has exactly one vector then this set must be linearly dependent. (False)

h) The sum of an upper triangular matrix and a lower triangular matrix is a diagonal matrix (False).

---

2. (1 mark each). Select one of the alternatives from the following questions as your answer.

a) Trace of the matrix $A = \begin{bmatrix} -6 & 8 \\ 2 & 9 \end{bmatrix}$ is:

(i) -3 (ii) 3 (iii) 15

b) Which of the following is a linear combination formed by the vectors $x_1 = (3, -4, 4)$, $x_2 = (2, -3, 1)$ and $x_3 = (-1, 1, -3)$ is:

(i) $x_1 = x_2 - x_3$ (ii) $x_3 = x_1 + x_2$ (iii) $x_2 = 2x_1 + x_3$

c) The inverse of the matrix $A = \begin{bmatrix} -3 & 1 \\ 3 & 1 \end{bmatrix}$ is

(i) $-\frac{1}{6}\begin{bmatrix} -3 & 1 \\ 3 & 1 \end{bmatrix}$ (ii) $-\frac{1}{6}\begin{bmatrix} 1 & -1 \\ -3 & -3 \end{bmatrix}$ (iii) $\frac{1}{6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$

d) The zero vector space {0} has dimension:

(i) 0 (ii)1 (iii) 2

e) If $A = \begin{bmatrix} 0 & -2 & 4 \\ -1 & 7 & 3 \\ 5 & 9 & -3 \end{bmatrix}$, then the cofactor of a_{22} is:

(i) 7 (ii) 20 (iii) -20

---

3. (3 marks). a. Find $u \times v$, where $u = (0,1,-3)$ and $v = (1,-1,2)$.

b. Show that the cross product in (a) is orthogonal to u.

$$u \times v = \begin{vmatrix} i & j & k \\ 0 & 1 & -3 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= -i - 3j - k$$

b. $u \cdot (u \times v) = (0,1,-3) \cdot (-1,-3,-1) = 0$

4. (2 marks) . If A= $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 3 & 2 \\ 4 & 2 & 2 \end{bmatrix}$ and $det(A) = 12$.

   Evaluate the determinant of the following matrix by using the above information.

$$B = \begin{bmatrix} 0 & -1 & 1 \\ 3 & 2 & 1 \\ 2 & 2 & 4 \end{bmatrix}.$$

- Matrix B resulted when we interchanged the columns of A twice.

  Hence $det\ (B) = -1 \cdot -1 \cdot det\ (A) = 12.$

5. (3 marks). If A= $\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $C = [2 \quad 3]$.

Calculate

i. $AB = \begin{bmatrix} 12 \\ 6 \end{bmatrix}$

ii. $(CB)^T = B^T C^T = [4 \quad 1] \begin{bmatrix} 2 \\ 3 \end{bmatrix} = [11]$

6. (4 marks). Solve the following system of linear equations by performing suitable row operations.

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

The Augmented matrix is:

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}, \quad (1)R_1 + R_2, \quad (-3)R_1 + R_3$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix}, (-1)R_2, \quad (10)R_2 + R_3$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{bmatrix}, (-\frac{1}{52})R_3$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix}, (-2)R_3 + R_1, \quad (5)R_3 + R_2$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}, \quad -R_2 + R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

The equivalent system of equations is:

$$x_1 = 3,$$
$$x_2 = 1,$$
$$x_3 = 2$$

Which is the solution of the system.